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Large Extra Dimensions and Decaying KK Recurrences

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Abstract

We suggest the possibility that in ADD type brane-world scenarios, the higher KK excitations of the graviton may decay to lower ones owing to a breakdown of the conservation of extra dimensional "momenta" and study its implications for astrophysics and cosmology. We give an explicit realization of this idea with a bulk scalar field Φ, whose nonzero KK modes acquire vacuum expectation values. This scenario helps to avoid constraints on large extra dimensions that come from gamma ray flux bounds in the direction of nearby supernovae as well as those coming from diffuse cosmological gamma ray background. It also relaxes the very stringent limits on reheat temperature of the universe in ADD models.

I. INTRODUCTION.

For many decades, starting with the pioneering work of Kaluza and Klein, the possibility that there may be extra hidden space dimensions has been considered seriously primarily for reasons such as the unification of gravity with other forces of nature. String theories provided a very compelling reason for extra space dimensions from purely theoretical considerations of conformal invariance and Lorentz invariance. There was a mini-boom of activity on the idea of extra space dimensions in the late 70's and early 80's following this realization. It seemed natural to assume that possible extra dimensions need not be as tiny as $l_{Planck} = M_{P\ell}^{-1} \sim 10^{-33}$ cm, which, in many ways, is the smallest distance possible, but rather any where in the region between this and the direct experimental upper bound from colliders i.e. $l \sim O(TeV^{-1}) \sim 10^{-18}$ cm.

The recent mega-boom of theoretical interest in the subject came in the wake of several papers which suggested the more radical possibility that the hidden space dimensions could be large [1,2], even as large as a millimeter [3], which is far above the 10^{-18} cm mentioned

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above without contradicting any known data. The class of theories where this possibility arises is one where the universe is assumed to have a brane-bulk picture, with gravitons but *not* the standard model particles propagating over a (much) larger extra dimension with $R = r_0 \times 10^{-3}$ cm [3–5]. The gravitational coupling to ordinary SM matter residing on the SM brane of size l is then diluted by a factor $f = [l/R]^d$ with d the number of extra dimensions. This "naturally" explains the weakness of gravity in the absence of a high fundamental energy scale. Also in these theories a crossover of the gravitational force law from the $1/r^2$ to $1/r^{(2+d)}$ should occur for $r < R$, adding a strong motivation to test Newton's law of gravity in an entirely new domain [6]. Choosing $d = 2$ allows satisfying the G(Newton) constraint:

$$\frac{l^2}{R} = l_{Planck} \sim 10^{-33} \quad cm \tag{1}$$

with $l \sim 10^{-18}$ cm consistent with Tevatron bounds and $R \sim 10^{-3}$ cm consistent with recent tests of the Newtonian $1/r^2$ law down to $O(100)$ micron distances [6].

These models [3] have interesting phenomenology, including enhanced neutrino cross-sections at high energy [7], and the possibility of producing microscopic Black Holes in accelerator and cosmic ray physics [8]. Also with the various SM fermions restricted to specific "domain walls" at locations $[v(i), u(i)]$ the masses mixing (CKM matrix elements) and potentially also the phases of the latter may be "explained" [9].

The large extra dimensions in this picture coexist with a low TeV level fundamental scale of nature which provides a new way to resolve the gauge hierarchy problem (i.e. why is $M_Z \simeq 10^{-16} M_{P\ell}$?). An added bonus of these theories is the possibility that string physics could be accessible to the present generation of colliders [10–13].

Despite these novel features, interest in such models has declined for various reasons, one of which is the lack of a convincing theoretical framework explaining the stability of the thick branes and the domain walls therein and another is the failure to find any hints in favor of this idea in gravity measurements [6] or in high energy collider experiments [13].

A third reason is perhaps the drastic change in our thinking about astrophysics and cosmology that is forced on us by large extra dimensions approach. For example the presence of a closely packed Kaluza Klein (KK) tower of massive graviton modes that is characteristic of these models, leads to a picture of early universe which is somewhat murky leaving unclear the nature of such issues as inflation [14] and baryogenesis [15]. The most unusual implication of this picture is that the universe cannot have a reheat temperature of more than few MeV; otherwise, the heavy KK modes that are produced overclose the Universe [3,16]. A Universe starting out at a temperature of a few MeV may barely have enough room for a successful BBN but it poses quite a challenge to understand the origin of matter [15]. Our goal is to address these as well as related issues of large extra dimension models (denoted as LED in this paper).

Strong upper bounds on R or l are derived from the following astrophysical considerations [17–19]. The emission of KK graviton recurrences from supernovae can

- (a) cause too quick a cooling of the latter and/or

- (b) the subsequent decay of the emitted KK recurences into photons may generate too many diffuse Gamma rays.

In the conventional ADD picture, massive KK modes of the gravitons only decay to photons, e^+e^-, etc., since their decays to lighter graviton KK modes are forbidden by the conservation of extra dimensional momentum. KK gravitons couple to brane matter with Planck mass suppression [3,11], and therefore, lifetimes for these decays are in the range of $\sim 10^7$ years for a KK mode with mass of 100 MeV. This coupled with very close spacing of the KK levels is the reason for strong constraints on the parameters of the model as we review below.

In this paper, we propose modified ADD type models where conservation of the extra dimensional momentum is violated allowing the massive KK modes of the gravitons to decay to other lighter graviton KK modes. There are then two competing decay modes of the massive KK gravitons: the conventional one to standard model particles such as electrons, neutrinos and photons that are confined to the brane and new ones to lighter KK modes of gravitons. For reasonable range of parameters, the new modes i.e. $g_{KK} \to g_{KK,1} + g_{KK,2}$ can dominate. In that case, it has the following kinds of effects: (i) it can prevent in the simplest manner possible overclosure of the universe by stable KK's relics, avoiding which otherwise requires some pattern of expanding and shrinking dimensions and very late inflation; (ii) it also can help to eliminate one class of supernova bounds that arise from the diffuse gamma ray background caused by the decays of the KK halo of neutron stars and (iii) finally, it can also relax the bounds that arise from cosmic diffuse gamma ray background arising from the decay of the graviton KK modes produced in the early universe. It however does not seem to affect the bounds that arise from supernova cooling by emission of KK modes.

This paper is organized as follows: in sec. 2, we review in a crude and intuitive manner how the cosmological bounds arise in the LED models; in sec. 3, we discuss the way the astrophysical bounds (a) and (b) above arise; in sec. 4 we review the argument forbidding the inter KK decays in the case of flat manifold compactification and note an amusing generalization for all compact manifolds with "KK Recurrences" transforming as any representations of the underlying (compact) Lie-algebra. In sec. 5, we show how the spontaneous breakdown of extra dimensional momentum conservation allows the gravitational decays of the higher KK modes of the graviton and estimate the new decay rates for various scenarios for the breakdown of the extra dimensional momentum; in section 6, we show how this new decay mode affects the astrophysical and the cosmological bounds. Finally, section 7 is devoted to discuss how the KK modes of an scalar field may pick up non zero vacuum expectation values. In particular, we show how a single scalar KK mode with nonzero extra dimensional momentum induces nonzero vev for a whole tower of modes, leading to what we call a "condensate tower". In section 8, we conclude with a summary of the results.

II. COSMOLOGICAL BOUNDS ON THE SIZE OF EXTRA DIMENSIONS

Any primordial abundance of KK gravitons recurrences can be diluted by inflation. However, as noted in [3,16] subsequent reheating even to temperatures of a few MeV will regenerate for extra dimension size of a millimeter a density of essentially stable KK modes of the graviton with masses $\leq T_R$, where T_R is the reheat temperature.

In order to see very crudely how the cosmological constraints arise, let us make the most conservative estimate for the emission rate of the g_{KK} modes from the collisions of the

standard model particles during the Hubble time $dt_H \sim -2dT \frac{M_{P\ell}}{\sqrt{g_*}T^3}$. Typical rates for g_{KK} emission from these collisions for each KK mode are:

$$R_{g_{KK}} \sim \frac{T^3}{M_{P\ell}^2}. \tag{2}$$

But at a given temperature, all KK modes upto T are produced. Thus in the Hubble time dt_H, the total number of g_{KK} modes can be estimated to be

$$N_{g_{KK}} \sim \int dt_H R_{g_{KK}} (TR)^2 \sim \frac{2}{3} \frac{T_R^3 R^2}{\sqrt{g_*} M_{P\ell}} = \frac{2}{3} \frac{M_{P\ell}\, T_R^3}{\sqrt{g_*} M_*^4}. \tag{3}$$

Note that most of the particles have masses close to the reheat temperature since the production rate goes down very sharply with temperature. Since these particles are stable, they survive all the way to today and therefore their number density today is roughly $\sim N_{g_{KK}} T_0^3$. Their contribution to energy density today can be estimated to be

$$\rho_0 \simeq \frac{2}{3} \frac{M_{P\ell}\, T_0^3 T_R^4}{\sqrt{g_*} M_*^4} \tag{4}$$

since modes close to T_R dominate as already mentioned. Setting $\rho \leq \rho_{cr}$, we get $T_R \sim$ 1 MeV [3,16]. We have summarized the essential points in this argument. For a more detailed analysis and the resulting refined bound, we refer to the paper [16]. We have neglected purely gravitational mechanisms for g_{KK} production, which would only enhance the $N_{g_{KK}}$ and strengthen the bound.

III. THE ASTROPHYSICAL BOUNDS

We first note that for the case of two extra dimensions, the number of KK modes with masses $< m$ strongly increases with m,

$$n_{(KK;\ m(KK)<m)} \sim [m \cdot R]^2 \sim r_0^2 [m/MeV]^2 \cdot 2.5 \times 10^{15}\ . \tag{5}$$

In cosmological or astrophysical settings where the temperature exceeds m, all these modes will be excited. We can now see the origin of the astrophysical bounds due (a) to KK emission and (b) to possible subsequent electromagnetic decays of the latter as follows:

(a) During the first few seconds of supernova collapse, the temperature of the collapsing core has a temperature $T = y \times 10$ MeV with $y \sim 1-5$. The spectrum of neutrinos from SN 1987a indicates the O(5) MeV temperature of the more extended, cooler, "neutrino sphere". To simplify we assume that roughly $N \simeq (RT)^2 = (r_0 \cdot y)^2 \cdot (2.5) \times 10^{17}$ KK 's with masses $<$ 30 MeV are emitted at the same rate as massless gravitons. They are emitted from relativistic particles inside the SN at a rate

$$(\tau)^{-1} \simeq T^3/M_{P\ell}^2 \simeq (1.6) \cdot y^3\ 10^{-18}\ \mathrm{sec}^{-1}\ . \tag{6}$$

Hence the energy of such particle (neutrinos, electrons/ positrons or photons) dissipates N times faster i.e during :

$$\Delta t \simeq \tau/N \simeq 2.5 \text{ sec} \cdot [r_0]^{-2} \cdot [y]^{-5} \ . \tag{7}$$

For $r_0 \geq 0.1$ and $y \simeq 3$, the emission of KK's yields cooling times $\Delta t \sim 10$ sec, commensurate with the observed duration of the SN 1987a neutrino pulses. Hence $R > 10^{-4}$ cm is excluded as the observed pulses would be shorter and weaker. Careful calculations by various groups [18] yield $R \leq 0.7 \times 10^{-4}$ cm. This bound relies only on the emission of the KK recurrences, not on possible radiative decays of the later.

(b) Turning now to the second class of supernova bounds [17], it arises as follows: for $r_0 < O(1)$ only a fraction ($< 30\%$) of the 10^{53} erg gravitational collapse energy in supernovas is emitted via KK recurrences. The mildly relativistic KK 's towards the upper end of the mass spectrum of those emitted from the supernova survive the cooling process thanks to their enhanced number and form a halo around the remnant neutron star. They decay at the same rate as in Eq. (6) with $T \simeq E \simeq m \simeq y \times 10$ MeV. Taking then $y \sim 3$, as above, we find a radiative KK decay rate: $\tau(R)^{-1} \sim 8 \times 10^{-18}$ sec^{-1} which is independent of the size of the extra dimensions. If the SN is at a distance of k kilo-Parsecs, the fraction of KK's decaying en-route is

$$f \simeq 0.4 \ k \times 10^{-7}. \tag{8}$$

Unlike for putative decays of light extreme [22] -relativistic neutrinos, photon from decaying mildly relativistic KK's will not appear shortly after the neutrino bursts and will not come precisely from the Supernova's direction. Thus the strict limits on Gamma/X-rays appearing shortly after the neutrino burst used to strongly limit electromagnetic decays of neutrinos are not useful here.

However Raffelt and Hannestad [17] have correctly noted that past supernovae collectively produce a "diffuse gamma ray background" at E $\sim$ 30 MeV exceeding experimental upper bounds- even if only a minute fraction $\sim 10^{-5}$ of the collapse energy is emitted via KK's.

The following crude estimates are no substitute for the careful, original work, yet may be of some interest for an overall understanding of the reason for the limit. The total observed electromagnetic "diffuse" energy in a $\sim$ 10 MeV interval around 30 MeV that can $\simeq \ 10^{-8}$ eV/cm^3 which is roughly 10^{-10} times the baryonic matter density which constitutes 1% of the critical density[1]. To satisfy the bounds on the diffuse 30 MeV gammas only 10^{-10} (!) of the baryonic rest mass can convert into such photons via emission of KK's with masses $\sim$ 30 MeV from SN's and the subsequent photonic decay of the latter.

To simplify and minimize red-shift and evolution effects consider only supernovae at less than 10^6 Kiloparsecs. From Eq. (8), we find that $\sim 4\%$ of the KK's decay over this distance; hence the total energy emitted as "heavy" i.e $\sim$ 30 MeV KK's should be less than $(2.5) \times 10^{-9}$ of the baryonic rest mass.

Assume that type II supernovae occurs at $\sim 1/30$ years per average galaxy with mass $\sim \ 10^{12} \ m_\odot$, in the above $z < 1/3$ range i.e 10^{-4} of all stars collapse over the lifetime of the Universe. In each collapse, approximately $\sim \ .25 \ m_\odot$ of gravitational energy is emitted out

[1]This bound gets further strengthened to about $\leq 10^{-9}$ eV/cm^3 if one uses the bound deduced [20] from the EGRET data [21] observations and extrapolate it according to the E^{-2} law.

of which about $\sim\ .1\ m_\odot$ can be emitted in the form of "massive" KK's without violating the first "cooling" bound. Thus $10^{-5} \times 0.04 = 4 \times 10^{-7}$ of the baryons rest mass could appear as 30 MeV gammas ! This exceeds the upper bound of 10^{-10} by 3.5 orders of magnitude unless the bulk dimension

$$R < 1.610^{-5} \quad \text{cm} \tag{9}$$

which is 70 times smaller than the first bound.

In any Type II supernova we expect $\sim .6\ m_\odot$ of iron to be ejected i.e six times more than the energy in massive KK's. The cosmological iron abundance of 10^{-3} of baryonic density thus implies that 1.6×10^{-4} (16 times more than in the previous estimate !) of the baryonic rest mass converts to massive KK's in all supernovae collapses. This requires $R < 4 \times 10^{-6}$ cm which is four times more restrictive than (9).

The above bounds push the value of the size of the extra dimension R far below what may be seen in direct gravitational measurement searches. Also Eq. (1) implies then that the thickness of the standard model brane is bound by

$$l < 10^{-19}\ \text{cm} \simeq (100\ \text{TeV})^{-1}\ ; \tag{10}$$

which is beyond the LHC range.

IV. FORBIDDENNESS OF INTRA-KK DECAYS IN THE BULK AND GENERALIZATIONS.

A basic rule of quantum mechanics is that corresponding to every noncompact space-like direction, there is a conserved quantum number which is familiar to us as momentum. When space is compactified (like the case of a particle in a box), the values of the momenta become discrete but the conservation law still holds. In the case of compact extra space dimensions, processes involving particles that live in the higher dimensions must conserve the corresponding discrete momentum quantum number. For example, in an allowed process $KK(k_1) + KK(k_2) \to KK(k_3) + KK(k_4)$, we must satisfy the condition $k_1 + k_2 = k_3 + k_4$. Note that conservation of extra dimensional momentum forbids all inter KK decays. This can be seen for the case of two dimensions as follows: suppose a particle of extra-D momentum $\vec{n}$ decays to two particle with extra momentum $(\vec{n}_1, \vec{n}_2)$. Then extra-D momentum conservation implies that $\vec{n} = \vec{n}_1 + \vec{n}_2$. The mass of the decaying particle, if it has no bare mass term, is given by $m_{KK}() = \sqrt{(\vec{n}_1 + \vec{n}_2)^2} \cdot R^{-1}$. Conservation of extra dimension momentum otgether with that of ordinary momentum then implies that the initial decaying mass equals the sum of the final masses leaving a vanishing (ordinary 3 dim.) phase space for the decay. The decay is therefore kinematically forbidden.

If the compactification is curved rather than flat, one can define analogs of extra dimensional momenta (called charges) and only bulk transitions conserving these charges will be allowed. Amusingly enough, this persists in even more restrictive form in the general case when the underlying internal group is any semi-simple compact Lie group and the extra dimensions manifold are any coset space. The KK particles are then invariantly characterized in the extra dimensions by a representation R of the group G to which they belong. The mass corresponding to the (internal) compact part of the Laplacian will be proportional to

$C_2(R)$ the Quadratic Casimir operator for the representation. The decay $A \to B + C$ with A, B and C in representations $R(A)$, $R(B)$ and $R(C)$ can occur only if the direct product $R(C) \otimes R(B) = \sum_i R(i)$ includes $R(A)$.

One can then show that $C_2(C) + C_2(B) > C_2(A)$ implying that $m(A) < m(B) + m(C)$ and any allowed intra KK decay will always be below threshold[2]. This can be motivated by verifying it as an algebraic identity for all representations of $SU(2)$:

$$(j.(j+1))^{1/2} + (k.(k+1))^{1/2} > ((j+k) + j + k + 1)^{1/2} \ ; \tag{11}$$

and also note the similarity to the triangular inequality which holds for convex internal spaces (or compact Lie groups).

V. SPONTANEOUS BREAKING OF THE TRANSLATIONAL INVARIANCE IN THE COMPACT DIMENSIONS AND DECAY OF KK GRAVITONS

In this section, we propose a model where the graviton KK modes are unstable and appear to provide a way around some of the astrophysical and cosmological constraints. Our proposal is to break the translation invariance along the hidden spacelike fifth and sixth directions while keeping in tact both Lorentz and translation invariance in the familiar 3+1 space time. In such a theory, the inequality in Eq. (11) will not be obeyed and therefore the heavier KK modes of gravitons can decay. One could in principle extend this also to extra dimensions with curvature; but here we focus only on flat extra dimensions in the spirit of the ADD model where the presence of such a spontaneous breaking, which leads to breakdown of the KK momentum conservation equation $\vec{n}_A + \vec{n}_B = \vec{n}_C$ in a particular reaction. As a result for any KK mode of the graviton, decays such as $A \to B + C$ are no more kinematically forbidden.

To implement our proposal, we take the usual brane-world scenarios where the standard model particles and forces are all confined to a 3-brane and gravity is in the bulk. We add a new scalar field $\phi(x, \vec{y})$ in the bulk; here x stands for the familiar space-time and $\vec{y} = (y_1, y_2)$ denotes the two hidden space dimensions. This bulk field can be expanded in terms of its Fourier components, which are fields that depend only on x and are the effective fields on the brane:

$$\Phi(x, y) \ = \ \sum_{n_1, n_2} \frac{1}{2\pi\, R}\ \phi_{\vec{n}}(x)\, e^{i\, \vec{n}\cdot\vec{y}/R} \ ; \tag{12}$$

for $\vec{n} = (n_1, n_2)$. We then consider an appropriate Higgs potential for the bulk field such that KK modes of ϕ, upto a large number, acquire vacuum expectation values,

$$\langle \phi_{\vec{n}} \rangle \ = \ v_{\vec{n}} \ . \tag{13}$$

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[2]We will not reproduce here the simple proof of this relation which has been kindly provided to us by Prof. Bernstein of the Math. Dept. in Tel-Aviv Univ. It utilizes the Harish-Chandra formula for $C_2(R)$ in terms of the maximal weights of R and simple features of such weights such as the additivity upon direct product.

This can be done for example by choosing a negative mass term for the ϕ field. We defer the discussion of this to a subsequent section, where we give some explicit examples. Note that as mentioned earlier, this leaves the four dimensional Lorentz as well as translation symmetry unbroken.

Let us next postulate the existence of a coupling of the form

$$\mathcal{L}_I \;=\; \lambda\, M_*^4\, \Phi\, g_{AB}\, g_{CD}\, g_{EF}\, \mathcal{C}^{ABCDEF} \;. \tag{14}$$

This kind of g^3 coupling could arise from generalized coordinate transformation invariant higher dimensional interactions of the form $\int d^4x\, d^2y\, \sqrt{-G}\, T_A^A(x,y)$ after expansion with $g_{AB} = \eta_{AB} + M_*^{-2} h_{AB}$ upto third order in h, and with T as the vacuum energy. We will use, however, Eq. (14) as a toy model interaction for our analysis below.

At the level of the effective four dimensional theory we get the following couplings among KK modes:

$$\mathcal{L}_I \;=\; \lambda\, \frac{M_*^2}{M_{P\ell}^2}\, \phi_{\vec{n}_\phi}\, h_{\vec{n}_1}\, h_{\vec{n}_2}\, h_{\vec{n}_3}\, \mathcal{C} \;. \tag{15}$$

To see how one gets the coupling strength of order $\left(\frac{M_*}{M_{P\ell}}\right)^2$, note that while expanding the Φ field in terms of 4-D fields, we get a factor $\frac{1}{R}$ and we have $g_{AB} = \eta_{AB} + \frac{1}{M_*^2 R} h_{AB}$. Noting that integration over d^2y gives R^2 and using $M_*^4 R^2 = M_{P\ell}^2$, we get the strength of the interaction given in Eq. (15). Clearly in this equation we have $\vec{n}_\phi + \sum_i \vec{n}_i = 0$. Once the four dimensional scalar field acquire a vacuum, $\langle \phi_{\vec{n}_\phi} \rangle \neq 0$, this interaction leads to $g_{KK} \to g_{KK} + g_{KK}$ as noted before, with proper identification of h as g_{KK}.

We show in sec. 7 that once one $\vec{n}_\phi \neq 0$ mode of Φ has nonzero vev, a tower of $\phi_{\vec{n}}$'s acquire nonzero vev by induction. We will call this a "condensate tower". The condensate tower plays an important role in enhancing the decay rate of the higher KK modes. To calculate the decay rate of a high graviton mode g_{KK}, we first note that for each $\langle \phi_{\vec{n}_\phi} \rangle \neq 0$, there are approximately $(\mu_{KK} R)^2$ open channels which counts the number of ways the equation $\vec{n}_i - \vec{n}_{f1} - \vec{n}_{f2} = \vec{n}_\phi$ can be satisfied for a given $\vec{n}_\phi$, where $\vec{n}_{i,f}$ denote the quantum numbers of the initial and final state KK modes. Then there is the contribution from each of the different $\langle \phi_{\vec{n}_\phi} \rangle \neq 0$ in the condensate tower contributing to the decay. Since each condensate tower consists of approximately $(\mu_{KK} R)^2$ entries, this gives an extra factor $(\mu_{KK} R)^2$ in the decay rate (assuming of course that the vevs are of same order. Combining all these, we get

$$\Gamma^{TOT}_{\vec{n},\; g\to gg} \;\simeq\; \frac{\lambda^2 \mu_{KK} \sum_{\vec{n}} \langle \phi_{\vec{n}} \rangle^2}{2\pi M_{P\ell}^2} \;, \tag{16}$$

and if all vevs are assumed to be same, we get

$$\Gamma^{TOT}_{\vec{n},\; g\to gg} \;\simeq\; \frac{\lambda^2 \mu_{KK}^3 R^2 \langle \phi_{\vec{n}} \rangle^2}{2\pi M_{P\ell}^2} \;=\; \frac{\lambda^2 \mu_{KK}^3 \langle \phi_{\vec{n}} \rangle^2}{2\pi M_*^4} \tag{17}$$

Compare this with the rate for a KK graviton decaying into photons which goes as $\Gamma_{g\to\gamma\gamma} \simeq \mu_{KK}^3 / M_{P\ell}^2$. It is clear that for a large enough vev, the inter KK decay channel dominates over the decay into standard model particles, since

$$\Gamma^{\rm total}_{\vec{n},\ g\to gg} \simeq \Gamma_{g\to\gamma\gamma}\left(\frac{\langle\phi_{\vec{n}}\rangle M_{P\ell}}{M_*^2}\right)^2 \ . \qquad (18)$$

For instance, if $\langle\phi_{\vec{n}}\rangle \sim M_* \sim$ TeV, we get an enhancement by at least a factor of 10^{30}. Thus, the KK graviton will predominantly decay into lighter KK gravitons.

VI. IMPLICATIONS OF THE MODIFIED ADD MODELS

Neutron star/supenovae limits We note from Eq. (18) that for reasonable values of $\langle\phi_{\vec{n}}\rangle$ of order of a TeV, only a tiny fraction of the ~ 30 MeV mass KK gravitons produced in old supernova could latter decay into photons and contribute to the diffuse gamma ray background. Naivlely taking the branching ratio in Eq. (18), one can see that at most a fraction $f \simeq 10^{-10}$ of the expelled KK modes would decay into visible channels. As noted in Ref. [17], the lower bound on the value of the fundamental scale M_* of LED models goes like $f^{1/4}$. In the absence of the new inter KK decay modes, $f \simeq 1$. Now that the fraction decaying to gamma rays is reduced, the lower bound on M_* is also reduced by $10^{-2.5}$ leading to $M_* \sim 1$ TeV.

Moreover, for EGRET's direct observation towards nearby neutron stars, we should notice that one has to take into consideration that the life time of a 30 MeV KK graviton would be much shorter than the one estimated in the usual case with only visible decay channels. Now one rather gets $\tau \lesssim 10^8$ sec for the above choice of parameters (the value considered for the bounds in [17] was about 10^{18} sec). This drastically reduces the present day source strength of MeV gamma rays by a factor of $exp(-t_{NS}/\tau)$, where $t_{NS} \sim 10^6$ yrs is a typical age of a neutron star. Thus, the present day source density of massive graviton KK modes should be many many orders of magnitude smaller than in the case with conservation of extra dimensional momenta. This makes the contribution to gamma emission by the nearby neutron stars negligible even for the closest such objects. This implies that the bounds on the size of extra dimensions obtained from the direct observations from supernova remnants derived in Ref. [17] do not apply.

Neutron star heating

It was suggested in Ref. [17], that the halo of heavy KK modes that should surrounds a neutron star remnant could heat up the star by the photons, e^+e^- pairs, and other decay products of the KK gravitons that continuously hit the star. In our model, with the dominance of the inter KK decay channels, most of the heavy KK gravitons would decay into lighter (relativistic) KK gravitons, which would then escape from the star, thus, giving no contribution to the heating process.

Cosmology bounds

As far as the upper bound on reheat temperature from cosmology is concerned, the situation is also relaxed. The analysis is somewhat more involved than the case of diffuse gamma ray background. Here we briefly summarize the salient points that make a difference in this analysis.

The production rate for the gravitational KK mode with mass μ_{KK} at a temperature T is roughly given by $R_{G_{KK}} \approx \frac{T^3}{M_{P\ell}^2}$. In previous discussions of this problem, the modes were

assumed to be stable. Since in our framework, the KK modes are unstable, we have to see what their lifetimes are. As mentioned before, when all KK modes of the ϕ field acquire similar vevs (we will show later this to be the case), the decay width of a mode with mass μ_{KK} is given by:

$$\Gamma^{tot}_{g_{KK}} \simeq \frac{\mu^3_{KK}\langle\phi_{\vec{n}}\rangle^2}{2\pi M^4_*} \ . \tag{19}$$

For $\mu_{KK} \simeq$ MeV and $M_* \simeq$ TeV, we get the life time of such modes to be of order 10^{-9} sec. which is far less than the age of the universe at 1 MeV. In fact the decay rate exceeds the Hubble expansion rate at any temperature above this. What this means is that above one MeV (the temperature at which BBN takes place), all heavy KK modes produced decay instantaneously to the lowest KK modes. These modes will redshift as the universe cools to the present temperature leading to their contribution to present energy density ρ_0 to be:

$$\rho_0 \simeq \frac{2}{3}\frac{M_{P\ell}\ T^3_R T^4_0}{\sqrt{g_*}M^4_*} \ ; \tag{20}$$

where T_0 is the present temperature of the universe. Comparing this with the expression in Eq. (4), we see that it implies that $T_R \leq 10$ MeV. Recall that a similar crude calculation without the KK decay led to a value of $T_R \simeq 1$ MeV. Thus we are able to relax the bound on the reheat temperature.

This calculation is very crude and is meant to illustrate the point. We expect that after the calculation is done more carefully, the value of T_R is likely to increase somewhat. It is also interesting to note that the reheat temperature T_R scales like $(M_*)^{4/3}$; thus an increase in M_* by a factor of ≈ 5 will increase the upper limit on T_R to a 100 MeV.

We also note that the KK recurrence decay mechanism discussed here does not seem to affect the constraints on large extra dimensions from supernova emission effects.

VII. INDUCING VEVS FOR SCALAR KK MODES

There are many ways a KK mode of a scalar field, ϕ, could pick up a nontrivial vacuum expectation value. Here we would like to comment some possibilities.

$\lambda\phi^4$ couplings.-

The simplest way to give a vev to the scalar field, in 6D, is to have a potential of the form

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4M^2_*}\phi^4 \ . \tag{21}$$

with $\mu^2 > 0$. There is always a simple solution to above equation, which is a y-independent vacuum (a true zero mode), given by the minimum of the potential $\partial V(\phi)/\partial\phi = 0$ that is given by $\langle\phi\rangle = \mu M/\sqrt{\lambda}$. As already mentioned, in the effective four dimensional theory, the above vev is larger, since

$$\langle\phi_0\rangle = \langle\phi\rangle R = (MR)\frac{\mu}{\sqrt{\lambda}} = \left(\frac{M_{P\ell}}{M_*}\right)\frac{\mu}{\sqrt{\lambda}} \;. \tag{22}$$

Therefore, for an order one λ, $\langle\phi_0\rangle$ can naturally be many orders of magnitude larger than μ. For our purpose, we will choose μ of order $1/R$ or less so that we get a vev only for the zero mode and a vev of order M_*. Physically, this is a consequence of the highly suppressed ϕ^4 effective coupling in 4D. Such a vev, however, being only for the zero mode, does not break translational invariance in extra dimensions and is therefore not useful for the new phenomena we discuss.

There are however other vacuum configurations that are allowed for the potential, where higher KK modes do get vevs. The configuration of such vacua is in general highly non trivial, at least from the point of view of the KK modes, as a perturbative analysis of the potential can show. To simplify the arguments let us just consider a simplest 5D model with a similar potential. The generalization to 6D should is straightforward. By integrating out the extra dimension in the action, one gets the effective potential in 4-D to be

$$V(\phi_n) = -\frac{1}{2}\mu^2\phi_0^2 - \sum_{n=1}^{\infty}\mu_n^2\phi_n\phi_{-n} + \frac{\tilde{\lambda}}{4}\sum_{k,\ell,m,n=-\infty}^{\infty}\delta_{k+\ell+n+m,0}\,\phi_k\phi_\ell\phi_n\phi_m \;; \tag{23}$$

where $\mu_n^2 = \mu^2 - n^2\mu_c^2$, with $\mu_c = R^{-1}$ the compactification scale, and $\tilde{\lambda} = \lambda/MR$.

Now, we notice that depending on the value of μ^2, there can be a set of modes for which the mass term, $-\mu_n^2$, is negative This will happen for modes starting with the zero mode and continuing all the way up to $n \simeq \mu R$. At a first glance, motivated by what happens in four dimensional theories, one could be tempted to believe that only modes in that particular set will pick up vevs. However, although it is correct that there will be solutions for the vacua with non zero KK vevs upto mode μR, it so happens that such configurations induce an infinite number of other modes to have nonzero vev, even though they may have their $\mu_n^2 > 0$, as long as it gives a vev to a mode with nonzero extra-D momentum.

To clarify our claim, let us assume, for instance, that all modes but $\phi_{\pm n}$ are zero. Next we observe that in the potential there is a coupling of the form $\phi_{\pm 3n}\phi_{\mp n}^3$ which contributes nontrivially to the minimization condition for $\phi_{\pm 3n}$. Indeed one gets $\partial V/\partial\phi_{\pm 3n} \propto (\phi_{\mp n})^3 \neq 0$. Hence, one can conclude that $\phi_{\pm 3n} \neq 0$. Once the mode ϕ_{3n} has nonzero vev, via the coupling, $\phi_{\mp 5n}\phi_{\pm 3n}{\phi_{\pm n}}^2$, which contributes to the minimization condition for $\phi_{\pm 5n}$, this will induce a vev for $\phi_{\pm 5n} \neq 0$. In a similar way one can proof that all $\phi_{(2k+1)n}$, for every k will have nonzero vev, just by noticing that there are always some couplings of the form $\phi_{(2k+1)n}\phi_{(2\ell+1)n}\phi_{(2m+1)n}\phi_{-[2(k+\ell+m)+3]n}$, involving all non-null vev modes. Therefore, barring miraculous cancellation happens among all those contributions, one is led to the conclusion that typically the vacuum will have an infinite number of modes picking up a vev, giving rise to the condensate tower. Moreover, one can see using this argument that three classes of vacua are in general possible: (i) those containing only modes with odd indices; (ii) those containing only modes with even indices; or (iii) with both classes of modes having non zero vevs.

By going to six dimension our conclusion will not change except that we must now include a second independent index, which follows the same rules of conservation that controls the way the mixing happens. The nonzero vevs in this case will form a lattice.

One can also show that all these new vevs correspond to physical minima i.e. the second derivative are positive. This condition for the minimum reads

$$\frac{\partial^2 V}{\partial\phi_n\partial\phi_{-n}} = -\mu_n^2 + 3\tilde{\lambda}\left[\phi_0^2 + 2\sum_{k=1}^{\infty}\phi_k\phi_{-k}\right] \ . \tag{24}$$

As the term between brackets is common to all values of n, one has that only the very first equation for the zero mode, $n = 0$, would be relevant to insure that any solution is a real minimum. Moreover, from the same expressions we see that the splitting among the physical masses of the KK modes would be still given by the compactification scale, μ_c.

Nambu-Goldstone modes:

An immediate implication of spontaneous breaking of the extra dimensional momentum is that it leads to the existence of massless scalar modes. This can be illustrated using a one dimensional example. Note that the Lagrangian involving the modes of the scalar field is invariant under the transformation $\phi_n \to e^{in\theta}\phi_n$. As a result when ϕ_n acquires a nonzero vev, there is a massless Nambu-Goldstone mode χ. To see the expression for χ, let us take the five dimensional case and express the field $\phi_n = \rho_n e^{i\chi_n/v_n}$. One then has:

$$\chi \ = \ N_\chi[\sum_n n v_n \chi_n] \tag{25}$$

as the massless state. Here $N_\chi^{-1} \ = \ \sqrt{\sum_n n^2 v_n^2}$. This has an obvious generalization to the case of two dimensions.

If the scalar field is coupled to brane fields, then this global symmetry is broken and the massless mode is a pseudo-Goldstone boson. The loop corrections will induce a mass to this mode of order $m_\chi^2 \simeq \frac{h^2}{16\pi^2}M_*^2$

In either case, all our phenomenological considerations remain unchanged.

Bulk Topological Defects.-

Strictly speaking, a y-dependent vacuum configuration for the ϕ^4 potential is an exact (non perturbative) solution to the equation of motion, obtained directly from the six dimensional action, and having no dependence on our standard four dimensions, thus, it is given by the solution to

$$\nabla_\perp{}^2\phi + \mu^2\phi = \frac{\lambda}{4M_*^2}\phi^3 \ ; \tag{26}$$

where $\nabla_\perp = \partial_{y_1}^2 + \partial_{y_2}^2$. A particularly interesting solution is the well known case of domain walls configuration, which can be written as

$$\phi(y) = v \ \tanh(\vec{\alpha}\cdot\vec{y}) \ ; \tag{27}$$

with $2\alpha^2 = \mu^2$ and $v = \mu M_*/\sqrt{\lambda}$. There are, as is well known, other non trivial solutions of this defect-type. It is clear, from the Fourier expansion, that in the present case all KK modes will get a nontrivial vev, since

$$\langle\phi_n\rangle = \frac{1}{2\pi R}\int_0^{2\pi R}\int_0^{2\pi R} d^2y\,\phi(y)\,e^{-i\vec{n}\cdot\vec{y}/R} = vR\left[\frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} d^2x \tanh(R\vec{\alpha}\cdot\vec{x})\,e^{2\pi i(\vec{n}\cdot\vec{x})}\right] \neq 0 \ .$$

Notice that the last expression within brackets would be just a number for a given n. Thus, the overall value of the KK vevs would be just $M_* R\,\mu/\sqrt{\lambda}$.

Using hidden branes.-

Another possibility is to have a vacuum on some (hidden) brane shining into the bulk by generating nontrivial vevs for all KK modes [23] of the bulk scalar. The mechanism works whenever the bulk scalar field mixes with a brane field, χ, that has a vev, for instance through the coupling

$$-M_*\phi(y)\,\chi\,\delta(\vec{y}) \ ; \tag{28}$$

which introduces a point-like source for the ϕ vacuum. Assuming that ϕ has only a mass term in the bulk, one easily finds that Fourier modes satisfy

$$\langle\phi_n\rangle = \frac{M_* R\,\langle\chi\rangle}{2\pi\,[(\mu R)^2 + \vec{n}^2]} \ . \tag{29}$$

Therefore, for a large enough μ we see that many KK modes i.e. all those below the threshold for which $\vec{n}^2 \ll (\mu R)^2$, get approximately the same vev:

$$\langle\phi_n\rangle \simeq \frac{M_* R\,\langle\chi\rangle}{2\pi(\mu R)^2} \ . \tag{30}$$

Notice, however, that each individual vev in this case is quite suppressed since $\langle\chi\rangle < M_*$. Nevertheless their accumulated effect, due to its large number, can easily overcome such a suppression when contributing to KK graviton decay since there are about $(\mu_{KK} R)^2$ of such modes.

In the presence of the vevs for many KK modes of the scalar field, there would a be residual vacuum energy in the five dimensional theory. If it is too large, it would lead to a breakdown of the flatness assumption of the fifth direction. It is therefore important to have an estimate of the contribution of $\langle\phi_{\vec{n}}\rangle$ to the potential $V(\phi(x,y))$. This can be done by noting that $\langle\phi_{\vec{n}}\rangle \simeq$ TeV implies that $\langle V(\phi(x,y))\rangle \simeq (10^{-7}\ GeV)^6$, which is much less than M_*^6 at which value the six dimensional curvature effect should be important. We therefore believe that the assumption of flatness of the two extra dimensions is not unreasonable.

Another radical and interesting alternative is to have a lattice of defects (e.g. vortices or domain walls) of lattice size $\approx l$ filling up the extra two dimensions and have the KK modes of gravitons live on lattice sites. These defects can cause a breakdown of the extra-D momentum and allow the higher KK modes of gravitons to decay, leading to phenomena similar to what is discussed in the paper. This idea is presently under study by one of the authors (S.N.).

VIII. CONCLUSIONS

In conclusion, we have raised the possibility of a new class of extra dimensional models where the translation invariance along the fifth and sixth dimensions is broken by the vevs of the KK modes of a bulk scalar field. This allows the higher KK modes of the gravitons to

undergo fast decay to all the lower lying KK modes of the graviton. The decay branching ratio of the higher KK modes of the graviton to standard model particles such as photons is then reduced by a significant amount with very interesting cosmological and astrophysical implications. For example, it now allows for the size of the extra two dimensions to be in millimeter range (with the higher dimensional gravity scale in TeV range as required to solve the gauge hierarchy problem) without conflicting with several astrophysical and cosmological bounds. At present, we do not see how this mechanism will be of help in relaxing the bounds from the supernova emission rates. Nonetheless, this is a new and interesting possibility that in our opinion deserves consideration. This possibility should also have implications for collider signatures of the TeV scale gravity models, that will be the subject of a future investigation.

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